



Midas Margin Model

Version 1.0

External

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1.0 Midas margin model

1.1 Summary

The purpose of this document is to provide a brief description of the margin methodology used by Clara CCP system. The margin model is used to compute a margin for clearing portfolios of each clearing member. Clearing portfolios can be based on multiple currencies and can contain multiple assets. The methodology is based on a value-at-risk estimated by a Monte Carlo technique, where interdependencies between underlying assets are taken into account. The computation is performed in close to real time.

1.2 Introduction

A negative margin requirement in our notation will denote that additional collateral in cash or eligible securities needs to be posted. The Midas Margin model estimates the value-at-risk at the 99% level of a portfolio containing equity and derivatives positions. The margin will be an estimate of the p -quantile of the future portfolio value distribution ($p = 0.01$), over a time period of Δt assumed to be two days.

1.3 Overview of methodology

The total margin requirement is obtained from the 1%-quantile of the distribution of the value of the portfolio, i.e the portfolio comprising the member's clearing and collateral positions. Relying on this quantile corresponds to a likelihood of 1% that the portfolio value will become less than the required margin within the close-out period of the instruments held by the clearing member.

1.4 Assumptions

The model relies on the following base assumptions:

- Risk factors are the variables the Midas margin model uses to identify the market risk of portfolio positions. The normalized risk factors are assumed to have an independent normalized Student t -distribution. i.e a variance of unity and an expected value of zero, with six degrees of freedom.
- The base currency of the Midas Margin Model is configurable. Cleared instruments may be denominated in another currency and a clearing member may use instruments in a foreign currency to meet its margin requirement.

2.0 Methodology

This chapter describes the methodology and procedures that Midas applies to calculate the margin of a portfolio containing stocks and derivatives. The p-quantile of the portfolio ($p = 0.01$) will be estimated based on a Monte Carlo simulation. For the margin calculation, Midas uses real-time prices of the underlying instruments. For the valuation of derivatives positions, Midas relies on theoretical pricing models for derivatives.

2.1 Stock model

Midas assumes stock prices are modeled as:

$$S_{t+\Delta}^i = S_t^i (1 + r^i) \quad (1)$$

where t is the current time, $t + \Delta t > t$ is the end of the close-out period (for stock i), S_t^i is the price of stock i at time t , and r^i is the stochastic return of the stock during the period Δt .

Further, the return r^i is modeled as $r^i = \lambda^i w^i$ where λ^i is the margin volatility of i , and w^i is the standardized return. These two components are modeled separately. The margin volatility models the stochastic price variation in stock, whereas the standardized return models interdependencies between all stocks.

$$S_{t+\Delta}^i = S_t^i (1 + \lambda^i w^i) \quad (2)$$

Margin volatilities are adjusted by the number of days in the close-out period, i.e. Δt is implicitly reflected in λ^i . How to obtain the margin volatility is described in 2.2.

w^i is modeled through a risk factor approach, i.e.

$$w^i = \sum_{j=1}^k Z_j \beta_{ij} + \epsilon \sigma_i \delta_i \quad (3)$$

where Z_j and ϵ are risk factors. β_{ij} are stock-specific weights in the Z_j and are denoted exposures of stock i to risk factor j . They reflect the correlation structure. σ_i is a noise parameter and δ_i is a parameter which is either -1 or 1 depending on the total direction of the position in the stock (long or short). Formula (3) will be described more in detail in 2.3.

2.2 Margin volatility

The margin volatility represents the magnitude of stochastic variation in an underlying stock price (with respect to the formula described in chapter 4.1), for the expected time interval needed to close out the relevant position (e.g. in a default scenario) and for a given confidence level. The

margin volatility is defined as the margin rate divided by 2.566, which is the 99% quantile of a normalized Student t-distribution with six degrees of freedom. The margin rate is set for each underlying instrument eligible for clearing, i.e. including subscription rights, when applicable, foreign exchange, and interest rate instruments. Margin rates are derived from observed (historical) data such as observed price volatility, turnover and trade frequency, as well as qualitative information on the issuer.

2.3 Beta and sigma values

Midas assumes correlations between equity returns, based on an Exponentially Weighted Moving Average (EWMA) model for the time interval $\Delta t = 2$.

Since the number of equities \mathbf{n} in the clearing universe might be large, it is not always feasible to model correlations between instruments directly. In order to perform a computation, it makes sense to reduce the number of dimensions to $\mathbf{k} < \mathbf{n}$. Our goal is to model the \mathbf{n} returns w as combinations of k risk factors Z . Thus for stock i we define.

$$\tilde{w}^i = \sum_{j=1}^k Z_j \beta_{ij}$$

or, in vector notation, for all stocks.

$$\tilde{\mathbf{W}} = \mathbf{ZB}$$

where $\tilde{\mathbf{W}}$ is a vector of the returns \tilde{w} , $\tilde{\mathbf{W}} = [\tilde{w}^1 \dots \tilde{w}^n]$,

\mathbf{Z} is a vector of the risk factors Z , $\mathbf{Z} = [Z^1 \dots Z^k]$ and

\mathbf{B} is a matrix of the β weights, $\mathbf{B} = \begin{bmatrix} \beta_{11} & \dots & \dots & \beta_{1n} \\ \vdots & \diagdown & \diagdown & \vdots \\ \vdots & \diagdown & \diagdown & \vdots \\ \vdots & \diagdown & \diagdown & \vdots \\ \vdots & \diagdown & \diagdown & \vdots \\ \beta_{k1} & \dots & \dots & \beta_{kn} \end{bmatrix}$

We want the correlation matrix of $\tilde{\mathbf{W}}$ to be equal the correlation matrix obtained by the EWMA model (\mathbf{E}).

For \mathbf{n} risk factors, we could achieve this exact result by defining $\mathbf{B} = \mathbf{D}\mathbf{V}^T$, with \mathbf{D} a diagonal matrix containing the square root of the eigenvalues of \mathbf{E} and \mathbf{V} the matrix of eigenvectors of \mathbf{E} .

$$\text{var}(\tilde{\mathbf{W}}) = \text{var}(\mathbf{ZB}) = \mathbf{B}^T \text{var}(\mathbf{Z}) \mathbf{B} = \mathbf{B}^T \mathbf{B} = \mathbf{V} \mathbf{D}^2 \mathbf{V}^T = \mathbf{E}$$

To reduce the number of dimensions, from n to k , we will now restrict ourselves to the k first eigenvalues and eigenvectors when constructing B . The number k will be chosen to satisfy the following criterion: the first k eigenvalues shall explain α of the total variation of the

Normalized returns:

$$\frac{\sum_{j=1}^k e_j}{\sum_{j=1}^n e_j} = \alpha.$$

The level α will be selected by the CCP.

Using k instead of n factors we will move us away from the desired EWMA correlation matrix. In order to compensate for two of the consequences, we will introduce parameters σ_i and δ_i , as well as a risk factor ϵ to define

$$w^i = \sum_{j=1}^k Z_j \beta_{ij} + \epsilon \sigma_i \delta_i$$

Firstly, when reducing the dimension, the diagonal of $B^T B$ (corresponding to the variance) will be lower than 1. For each stock i , we now define

$$\sigma_i = \sqrt{1 - \sum_{j=1}^k \beta_{ij}^2}$$

This will ensure that w^i will have a variance of unity for each i .

$$\text{var}(w^i) = \text{var}\left(\sum_{j=1}^k \beta_{ij} Z_j + \epsilon \sigma_i \delta_i\right) = \sum_{j=1}^k \beta_{ij}^2 + \sigma_i^2 \delta_i^2 = \sum_{j=1}^k \beta_{ij}^2 + (1 - \sum_{j=1}^k \beta_{ij}^2) = 1$$

Secondly, after the reduction of dimension, the off diagonal elements (corresponding to correlations) of the EWMA matrix will no longer be precisely met. In order to mitigate this effect we will set the value of δ_i to be either -1 or 1 depending on the the total direction of the position in stock i , S_i , in the portfolio(long or short). One could thus also describe δ_i as the opposite sign of the partial derivative of instrument i with respect to the portfolio. For the correlation between the returns of stocks i and l , we thus obtain

$$\text{corr}(w^i, w^l) = \sum_{j=1}^k \beta_{ij} \beta_{lj} + \sigma_i \sigma_l \delta_i \delta_l$$

i.e. initial correlations in $B^T B$ will be moved up or down by $\sigma_i \sigma_l$.

Choosing δ_i as indicated, $\delta_i \delta_l$ corresponds to a move towards the worse i.e more conservative correlation of instrument i and l.

Position in S_i	Position in S_l	Worst move	Worst correlation
long	long	same direction	1
short	short	same direction	1
long	short	opposite direction	-1

Finally, the normalized return for each instrument i

$$w^i = \sum_{j=1}^k Z_j \beta_{ij} + \epsilon \sigma_i \delta_i \quad (3)$$

can also be given in vector notation for all instruments

$$\mathbf{W} = \mathbf{ZB} + \epsilon \boldsymbol{\sigma} \boldsymbol{\delta}$$

with $\boldsymbol{\sigma} \boldsymbol{\delta} = [\sigma_1 \delta_1, \dots, \sigma_n \delta_n]$

One way of thinking about the risk factors Z and ϵ is as follows: Risk factors Z are market risk factors, which represent market movement. The risk factor ϵ is a random noise, corresponding to an idiosyncratic movement of the instrument price, that cannot be explained by market risk factors.

Consequently formula (3) can be split-up in two parts, A and B:

$$\sum_{j=1}^k \underbrace{Z_j}_{\mathbf{A}} \beta_{ij} + \underbrace{\epsilon \sigma_i \delta_i}_{\mathbf{B}}$$

Part A, $\sum_{j=1}^k Z_j \beta_{ij}$, can be thought of as the "explained part" with β_{ij}^2 representing how much is explained by risk factor Z_j . Part B, $\epsilon\sigma_i\delta_i$ is the "unexplained part" with σ_i^2 representing, is explained by price variations idiosyncratic to the instrument.

Hence, for a given portfolio the correlation matrix reads:

$$\text{var}(\mathbf{W}) = \mathbf{B}^T \mathbf{B} + \sigma \delta^T \sigma \delta$$

For equities traded on less than 55 of the last 60 trading days, parameters β will be set to 0 and hence σ will be set to 1.

2.4 Models for derivatives

The potential price distribution of an instrument is considered to be a function of its risk factors, which are the stochastic drivers of the prices. The present value of equity instruments is observed directly in the market. For forwards, futures and options, the present value is derived from the price of the underlying instruments as well as additional parameters by theoretical pricing models. The general notation as defined below will be used throughout the following subsections:

- S_t^i is the price of stock i at time t
- K is the strike price of an option
- r_{rf} is the quoted risk-free rate for the applicable period. Discrete compounding is applied with a day-count convention of (ACT/360). The unit is the calendar year.
- r is the continuously compounded risk free rate $r = \log(1 + 365/360 r_{rf})$, the unit is the calendar year.
- T is the maturity (expiry) data and $\Delta T = T - t$ the time to maturity of the derivative instrument. The latter is measured in units of calendar years.
- μ_i is the Option Volatility (cf. section 2.6) of stock i. It is measured in units of market years (250 days).
- $\Phi()$ is the cumulative distribution function of the standard normal distribution. In the context of option pricing, the following two formulae are used

$$- d_1 = d_2 + \mu\sqrt{\Delta T}$$

$$- d_2 = \frac{\log\left(\frac{S_t}{K} + (r - \frac{\mu^2}{2})\Delta T\right)}{\mu\sqrt{\Delta T}}$$

2.5 Options

Options are priced using the Black and Scholes formula for European options. The same formula is also used for American options. Although the possibility of early exercise gives these options a higher value, this effect is deemed immaterial for margining purposes.

According to Black-Scholes, the price of an European call option is given by

$$C^t(S_t^i, K, r, \Delta T, \mu_i) = S_t^i \Phi(d_1) - e^{-r\Delta T} K \Phi(d_2) \quad (5)$$

The price of an European put option is given by

$$P^t(S_t^i, K, r, \Delta T, \mu_i) = e^{-r\Delta T} K \Phi(-d_2) - S_t^i \Phi(-d_1) \quad (6)$$

2.6 Option volatilities

The pricing formulae for options in section 4.6 require volatility as an input, denoted as μ_i . For Midas, the principle of Uncertain Volatility has been adopted, which assumes that the volatility of an instrument is unknown, but within a volatility range (see Avellaneda, 1995). Options are assumed to be monotone with respect to price, and a high and low price for an option can be estimated based on the endpoints of its volatility range, which we will denote the high and low volatility. The high (low) volatility will be used in the Black-Scholes formula whenever a derivative position is short (long). When margining a portfolio, Midas will therefore consider the highest (lowest) price estimate. When estimating the volatility range, Midas relies on two different methods depending on the availability of historical prices. If sufficiently many historical prices are available, the volatility range is determined on the basis of historical prices (cf. 2.7). If there are too few data points, the model will rely on default volatilities set by the CCP (cf. 2.8)

2.7 Volatilities based on historical prices

Midas estimates option volatilities relying on historical data using an Exponentially Weighted Moving Average (EWMA) model with a lambda of 0.94. In estimating high and low volatility, the estimates of the last 60 trading days are considered. The high and low volatilities are set as the highest and lowest estimate for this time period, multiplied by a factor of 1.25 and 0.75, respectively. This results in a conservative boundary on the volatility. This method of estimating option volatility is used if the underlying instrument has been traded for at least 55 of the last 60 trading days.

2.8 Default volatilities

If sufficient historical data is not available, pre-specified default values for the option volatilities are used. The default volatilities are defined based on the margin volatility of the underlying.

The high volatility is defined as

$$\lambda_{\text{high}} = \min(3, 1.25e^{3\mu^i} - 0.4) \quad (7)$$

The low volatility is defined as

$$\lambda_{\text{low}} = \min(0.5, \max(0.05, 1 - e^{-2\mu^i})) \quad (8)$$

3.0 Monte Carlo simulation

The Midas margin model uses a Monte Carlo simulation to estimate the p-quantiles for each portfolio. The number of simulations **m** depends on the time of day when the margin is calculated. For start-of-day calculations, which will form the basis for the daily margin requirement, Midas uses 100,000 simulations. For intraday calculations, 10,000 simulations are performed. Midas creates **m** possible market scenarios by sampling the k+1 risk factors Z and ϵ . By means of formula (3), **m** different market scenarios for w result, which again yield **m** different scenarios for stock prices based on the margin volatilities and formula (2). Using the derivatives formulae in sections 4.4 to 4.6, we can obtain portfolio values for each of the simulated scenarios. The estimate for the p-quantile of the portfolio is then given by the p*n highest value of the sampled portfolio values (e.g. $100000 * 0.01 = 1000$ for the startof-day calculation).

3.1 Example

Below we provide an example that will go through the steps of calculating a margin requirement. We start with portfolio (P):

INSTRUMENT	STL	IKEA	STL _{CALL}	NOK	SEK
Quantity	Q 1	Q 2	Q 3	-Q 4	-Q 5

STL is a Norwegian stock with price in NOK, IKEA is a Swedish stock with price in SEK, STL_{CALL} is a call option on STL, and SEK and NOK are cash legs in the respective currencies.

The portfolio value in NOK at time t:

$$P_t = Q_1 \text{STL}_t + Q_2 \text{IKEA}_t \text{FX}_t^{\text{SEK/NOK}} + Q_3 C(\text{STL}_t) - Q_4 - Q_5 \text{FX}_t^{\text{SEK/NOK}}$$

C() is the Black-Scholes pricing formula (5) in 4.6, and $\text{FX}_t^{\text{SEK/NOK}}$ is the SEK to NOK exchange rate at time t.

The margin requirement is the 1% quantile of the portfolio value at time $t + \Delta t$. The portfolio value in NOK at time $t + \Delta t$ is:

$$P_{t+\Delta t} = Q_1 STL_t(1 + r^{STL}) + Q_2 IKEA_t(1 + r^{IKEA})FX_t^{SEK/NOK}(1 + r^{FX}) + Q_3 C(STL_t(1 + r^{STL})) - Q_4 - Q_5 FX_t^{SEK/NOK}(1 + r^{FX})$$

The margin is calculated by creating m different scenarios, for each of which the portfolio value is computed. Finally, the $(0.01 * m)$ lowest value of all scenarios is taken to be the margin requirement. For each scenario, prices for STL, IKEA, FX SEK/NOK are simulated, where the prices are derived from the following formula.

$$S_{t+\Delta}^i = S_t^i(1 + r^i) = S_t^i(1 + \lambda^i w^i),$$

i.e using the methodology set out in 2.3

Here, we assume that we have reduced the dimension to 2, so $k = 2$. Given the values as shown in the matrix below, $Z_1, Z_2 \cdot \epsilon$ are simulated for each scenario:

	β_{1i}	β_{2i}	σ_i	δ_i
STL	β_{STL1}	β_{STL2}	σ_{STL}	δ_{STL}
IKEA	β_{IKEA1}	β_{IKEA2}	σ_{IKEA}	δ_{IKEA}
$FX^{SEK/NOK}$	β_{FX1}	β_{FX2}	σ_{FX}	δ_{FX}

Then w_i is given by formula (3)

$$w^i = \sum_{j=1}^k Z_j \beta_{ij} + \epsilon \sigma_i \delta_i$$

w_{STL}	$Z_1 \beta_{1STL} + Z_2 \beta_{2STL} + \epsilon \sigma_{STL} \delta_{STL}$
w_{IKEA}	$Z_1 \beta_{1IKEA} + Z_2 \beta_{2IKEA} + \epsilon \sigma_{IKEA} \delta_{IKEA}$
$w_{FX^{SEK/NOK}}$	$Z_1 \beta_{1FX} + Z_2 \beta_{2FX} + \epsilon \sigma_{FX} \delta_{FX}$

According to section 4.3, β_{1i}^2 shows how much of the move in the return is explained by risk factor Z_1 , and β_{2i}^2 shows how much of the move in the return is explained by risk factor Z_2 .

$\beta_{1i}^2 + \beta_{2i}^2$ is then the part explained by the Z's. σ_i^2 denotes how much of the movement is caused by the "unexplained part", or the part which is idiosyncratic to the instrument. The factors δ_i decide the direction to which the unexplained part moves. From the simulated instrument returns w , we can construct simulated prices

From the simulated instrument returns w , we can construct simulated prices

$STL_{t+\Delta t}$	$STL_t(1 + \lambda^{STL}_W^{STL})$
$IKEA_{t+\Delta t}$	$IKEA_t(1 + \lambda^{IKEA}_W^{IKEA})$
$FX_{t+\Delta t}^{SEK/NOK}$	$FX_t^{SEK/NOK}(1 + \lambda^{FX}_W^{FX})$

These can be used to determine the value of the portfolio for each scenario

$$P_{t+\Delta t} Q_1 STL_{t+\Delta t} + Q_2 IKEA_{t+\Delta t} FX_{t+\Delta t}^{SEK/NOK} + Q_3 C(STL_{t+\Delta t}) - Q_4 - Q_5 FX_{t+\Delta t}^{SEK/NOK}$$

After having computed all portfolio values for all scenarios, we obtain the margin requirement as the 1% quantile of the portfolio values.